Math 70 — 11.2 Solving Quadratic Equations using the Quadratic Formula — Lesson 29

11.3 Using the Discriminant to describe the number and type of solutions of a quadratic equation without solving it

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Objectives

1) Solve equations using the quadratic formula

2) Review simplifying square roots, if necessary.

3) Identify the discriminant

$D = b^2 - 4ac$

4) Review types of numbers

real rational
real irrational
imaginary
complex

5) Use discriminant to determine the number and type of solutions of a quadratic equation without solving it.

Math 70 11.2 & 11.3 Solving Equations by the Quadratic Formula (and using the Discriminant)

Objectives:

1) Solve quadratic equations using the quadratic formula. $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$
2) Calculate the discriminant $D=b^2-4ac$ for a quadratic equation.
    a. The discriminant does not have a square root.
    b. Use the discriminant to determine quickly if an equation can be factored.
3) Interpret the value of the discriminant to determine the number of real and complex solutions of a quadratic equation.
4) Compare and contrast the methods for solving quadratic equations

Background skills you should know:

a. simplifying square roots
b. real rational numbers
c. real irrational numbers
d. complex numbers a+bi

<u>Sets of Numbers</u>

Real numbers: Any number without an i.

Real rational numbers:

Can be written as a fraction of two integers

Decimal representation either terminates or repeats a pattern (of fixed length) infinitely

Real irrational numbers:

Cannot be written as a fraction of two integers

Decimal representation continues infinitely without pattern of fixed length.

Imaginary numbers: Any single term, with an i, bi.

Complex numbers: Two terms, one with an i and one without, a+bi.

Sign of the discriminant	Perfect Square	Number of solutions	Type of solutions
Positive	Perfect square*	2	Real, rational
	Not a perfect square	2	Real, irrational
Zero*	Doesn't matter	1	Real, rational
Negative	Doesn't matter	2	Complex

*If the discriminant is a positive perfect square or zero, the equation can be factored.

Method	Advantages	Disadvantages	Things to remember
Factoring with zero-product property $(x+e)(x-d)=0$ $x=-e,d$	Some equations factor easily and solve quickly. $x^2-6x-7=0$ $x^2-6x=0$ Can be used to solve higher-order equations which factor.	Some equations are difficult to factor and take a long time. $72x^2-41x-117=0$ Other equations cannot be factored at all. $x^2-6x+7=0$	The right-hand-side of the equation must be zero.
Square root property $x^2=n$ $x=\pm\sqrt{n}$	Fastest way to solve equations in the forms: $x^2=3$ $3x^2=12$ $(4x+1)^2=9$ $4(x-1)^2-8=0$		Isolate the square first. Must include $\pm$ when taking the square root of both sides. RHS should not be zero.
Completing the square followed by the square root property. $ax^2+bx+c=0$ $\left(x+\frac{b}{2}\right)^2=-\frac{c}{a}+\left(\frac{b}{2}\right)^2$	Can be used to solve any quadratic equation. Does not usually involve simplified complicated square roots. GC's MATH >frac makes the fraction calculations easy. Is used to create the quadratic formula.	The process has several steps which cannot be done out of order. There are more "steps" to describe the process. Fractions result when $a\neq 1$ or b is not even.	The leading coefficient must be 1, so divide entire equation by any leading coefficient that's not one. RHS should not be zero. Must add $\left(\frac{b}{2}\right)^2$ to both sides of the equation. The factor is always $\left(x+\frac{b}{2}\right)^2$
Quadratic formula $ax^2+bx+c=0$ $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$	Can be used to solve any quadratic equation.	Always requires simplifying a complicated square root. Must memorize the formula.	Equation must be fully simplified (no parentheses) and in standard form prior to using the formula. RHS must be equal to zero. Entire formula is divided by 2a, not just the radical.
Approximate solutions by graphing	Gives intuition and way to check work.	Likely to be approximate answers. Must calculate twice if there are two answers. Won't help if answers are complex.	Intersection method – two graphs, one for LHS and one for RHS. x-Intercept method – one graph of LHS-RHS or RHS-LHS.

Examples:

1) $3x^2 - 9x + 8 = 0$
   a. Solve by the QF.
   b. Identify the number and type of solutions using the previous work.
   c. Check solutions by graphing.

2) $6x^2 - 17x - 14 = 0$
   a. Solve by the QF.
   b. Identify the number and type of solutions using the previous work.

3) $25x^2 - 20x + 4 = 0$
   a. Solve by the QF.
   b. Identify the number and type of solutions using the previous work.

<u>Without</u> solving the equation, calculate only the discriminant and use it to determine the number and type of solutions.

4) $2x^2 - 4x = 3$

5) $m^2 - \frac{m}{2} + \frac{1}{16} = 0$

6) $\frac{x}{3} = -x^2 - 1$

Solve the problem.

7) An investment of $2000 grows to $2420 when compounded annually for two years. What is the interest rate?

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Process for solving by the Quadratic Formula (QF)

Memorize the quadratic formula
for $ax^2+bx+c=0$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

Step 1: Set equation $=0$ and write in standard form $ax^2+bx+c=0$.

Step 2: Identify the coefficients a, b, c.

Step 3: Substitute values of a, b, c into formula.

Step 4: Simplify result.

Process for Checking Solutions using graph on GC

The graph shows real solutions only. If answer has i, the graph can only confirm that the solutions have imaginary parts, but not the actual numerical values.

step 1: Set equation equal to zero

Step 2: Graph the expression using $y=$

step 3: Find the x-intercepts of graph using "zero" (or "root")

step 4: If necessary, calculate the approximate values of the solutions found by CTS or QF so they can be compared to solutions found in step 3.

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① $3x^2 - 9x + 8 = 0$ (2 complex) QF

$a = 3 \quad b = -9 \quad c = 8$

$$x = \frac{-(-9) \pm \sqrt{(-9)^2 - 4(3)(8)}}{2(3)}$$

$$x = \frac{9 \pm \sqrt{81 - 96}}{6}$$

$$x = \frac{9 \pm \sqrt{-15}}{6}$$

$$x = \frac{9}{6} \pm \frac{i\sqrt{15}}{6}$$

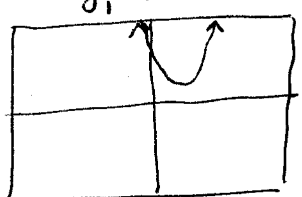
2 Complex solutions

$$\boxed{x = \frac{3}{2} \pm \frac{i\sqrt{15}}{6}}$$

a+bi form

Graph

$y_1 = 3x^2 - 9x + 8$



no x-ints

⇒ no real solutions

*2 We cannot check the numerical values of a complex number using a graph because it shows real numbers only

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Extra (Review)

(1) cont Check $x = \frac{3}{2} \pm i\frac{\sqrt{15}}{6}$ are solutions of $3x^2 - 9x + 8 = 0$

by evaluating the function.

$\boxed{Y=}$ $3x^2 - 9x + 8$

3/2 + [2nd] [.] √(15)/6 [STO>] $\underbrace{\text{[ALPHA] [STO>]}}$

2nd decimal pt is $i = \sqrt{-1}$

[ALPHA][STO>] is memory location X

```
3/2+i√(15)/6→X
   1.5+.6454972244i
```

[VARS] [▷] [Y VARS] 1. Function
1. Y_1

[(] $\underbrace{\text{[ALPHA] [STO>]}}_{\text{This is X}}$ [)] [ENTER]

This doesn't work for complex numbers ☹

ERROR: DATA TYPE.

$$= 3\left(\frac{3}{2} + i\frac{\sqrt{15}}{6}\right)^2 - 9\left(\frac{3}{2} + \frac{i\sqrt{15}}{6}\right) + 8$$

$$= 3\left(\frac{3}{2} + \frac{i\sqrt{15}}{6}\right)\left(\frac{3}{2} + \frac{i\sqrt{15}}{6}\right) - \frac{27}{2} - \frac{3i\sqrt{15}}{2} + 8$$

$$= 3\left(\frac{9}{4} + 2\cdot\frac{3}{2}\cdot\frac{i\sqrt{15}}{6} - \frac{15}{36}\right) - \frac{11}{2} - \frac{3}{2}i\sqrt{15}$$

$$= \frac{27}{4} + \frac{3}{2}i\sqrt{15} - \frac{5}{4} - \frac{11}{2} - \frac{3}{2}i\sqrt{15}$$

$$= 0$$ ☺

You can do it by hand

3/2 + [2nd] [.] √(15)/6 [STO>] $\underbrace{\text{[ALPHA] [STO]}}_{X}$

3 [ALPHA] [STO>] [x^2] − 9 [ALPHA] [STO>] + 8 [ENTER]

```
3/2+i√(15)/6→X
   1.5+.6454972244i
3X²-9X+8
                0
```

or you can type it into GC yourself (no y= menu)

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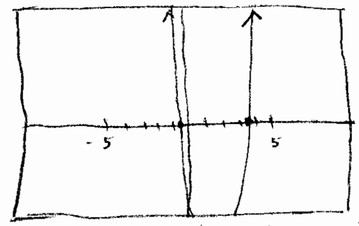
(2) $6x^2 - 17x - 14 = 0$ (2 rational)

Method 2: Quadratic Formula



2 x ints at $x = 7/2, -2/3$

$a = 6 \quad b = -17 \quad c = -14$

$$x = \frac{-(-17) \pm \sqrt{(-17)^2 - 4(6)(-14)}}{2(6)}$$

$$x = \frac{17 \pm \sqrt{625}}{12}$$

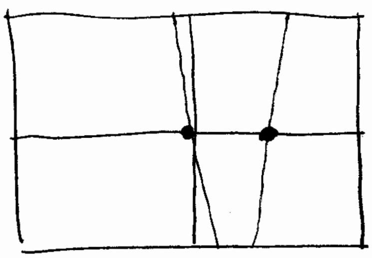
$$x = \frac{17 \pm 25}{12}$$

$$x = \frac{17+25}{12}, \frac{17-25}{12} = \boxed{\frac{7}{2}, \frac{-2}{3}}$$

2 real rational solutions

Graph

$y = 6x^2 - 17x - 14$



$\boxed{2nd}$ TRACE = CALC
2. zero

$x \approx -.6666667 \Rightarrow -.\overline{6} = \frac{-2}{3}$

$x \approx 3.5 \Rightarrow x = \frac{7}{2}$

Extra Review

9.) $6x^2-17x-14=0$ (2 rational) Method 1A: CTS factor out leading coef

$6\left(x^2-\frac{17}{6}x \qquad\right)=14$

$\#^2=\left(\frac{-17}{6}\cdot\frac{1}{2}\right)^2=\frac{289}{144}$

Use GC > frac

$6\left(x^2-\frac{17}{6}x+\frac{289}{144}\right)=14+6\cdot\frac{289}{144}$

$6\left(x-\frac{17}{12}\right)^2=\frac{625}{24}$

$\left(x-\frac{17}{12}\right)^2=\frac{625}{24(6)}$

$x-\frac{17}{12}=\pm\sqrt{\frac{625}{144}}$

$x=\frac{17}{12}\pm\frac{25}{12}$

$x=\frac{17}{12}+\frac{25}{12},\ \frac{17}{12}-\frac{25}{12}$

$x=\frac{42}{12},\ \frac{-8}{12}$

$\boxed{x=\frac{7}{2},\ \frac{-2}{3}}$

2 real rational solutions!
Could be factored!
$(2x-7)(3x+2)=0$

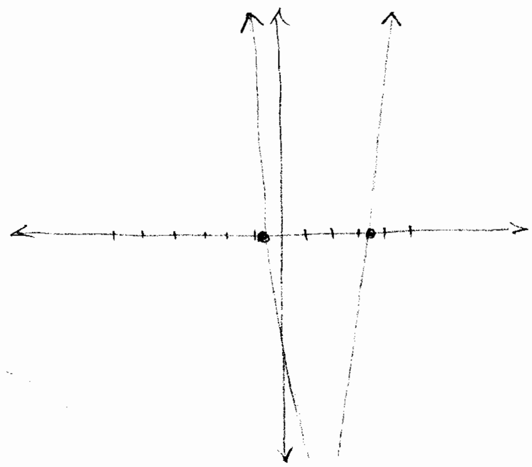
$x=\frac{-(-17)\pm\sqrt{(-17)^2-4(6)(-14)}}{2(6)}$

$x=\frac{17\pm\sqrt{289+336}}{12}$

$x=\frac{17}{12}\pm\frac{\sqrt{625}}{12}$

$x=\frac{17}{12}\pm\frac{25}{12}$

$\boxed{x=\frac{7}{2},\ \frac{-2}{3}}=3.5,\ -.\overline{6}$



$6x^2-17x-14=0$ (2 rational) Method 1B: CTS divide by leading coef

$\frac{6x^2}{6}-\frac{17x}{6}=\frac{14}{6}$

$x^2-\frac{17}{6}x=\frac{7}{3}$

CTS
$\#=\frac{-17}{6}\cdot\frac{1}{2}=\frac{-17}{12}$ ← factor
$\#^2=\left(\frac{-17}{12}\right)^2=\frac{289}{144}$ ← add to both sides

$x^2-\frac{17}{6}x+\frac{289}{64}=\frac{7}{3}+\frac{289}{144}$

$\boxed{\text{MATH}}$ > frac is your friend!

$\left(x-\frac{17}{12}\right)^2=\frac{625}{144}$

$x-\frac{17}{12}=\pm\frac{\sqrt{625}}{\sqrt{144}}$

$x=\frac{17}{12}\pm\frac{25}{12}$

$x=\frac{17}{12}+\frac{25}{12}=\frac{42}{12}=\boxed{\frac{7}{2}}$

$x=\frac{17}{12}-\frac{25}{12}=-\frac{8}{12}=\boxed{\frac{-2}{3}}$

3

$25x^2 - 20x + 4 = 0$ (1 rational)

$a = 25 \quad b = -20 \quad c = 4$

$$x = \frac{-(-20) \pm \sqrt{(-20)^2 - 4(25)(4)}}{2(25)}$$

$$x = \frac{20 \pm \sqrt{0}}{50}$$

$$\boxed{x = \frac{2}{5}}$$

one real rational solution

$25x^2 - 20x + 4 = 0$ (1 rational)

③ Extra Review

$25\left(x^2 - \frac{20}{25}x \qquad\right) = -4$

$25\left(x^2 - \frac{4}{5}x \qquad\right) = -4$

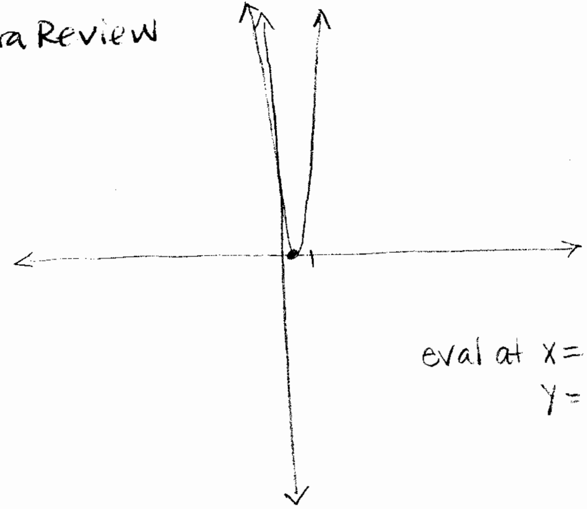
$\#^2 = \left(\frac{-4}{5}\cdot\frac{1}{2}\right)^2 = \left(\frac{-2}{5}\right)^2 = \frac{4}{25}$

$25\left(x^2 - \frac{4}{5}x + \frac{4}{25}\right) = -4 + 25\left(\frac{4}{25}\right)$

$25\left(x - \frac{2}{5}\right)^2 = 0$

$\left(x - \frac{2}{5}\right)^2 = 0$

$\boxed{x = \frac{2}{5}}$ ← one real rational solution



eval at $x = .4$
$y = 0$

Note: This can be factored to a better perfect square in the first step:
$(5x-2)^2 = 0.$

$25x^2 - 20x + 4 = 0$ (1 rational)

$\frac{25x^2}{25} - \frac{20x}{25} = \frac{-4}{25}$

$x^2 - \frac{4}{5}x = \frac{-4}{25}$

$\# = \frac{-4}{5}\cdot\frac{1}{2} = \frac{-4}{10} = \frac{-2}{5}$ ← factor

$\#^2 = \frac{4}{25}$ ← add to both sides

$x^2 - \frac{4}{5}x + \frac{4}{25} = \frac{-4}{25} + \frac{4}{25}$

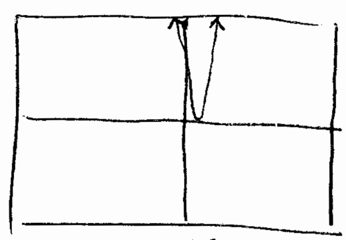
$\left(x - \frac{2}{5}\right)^2 = 0$

$x - \frac{2}{5} = \pm\sqrt{0}$

$x - \frac{2}{5} = 0$

$\boxed{x = \frac{2}{5}}$

Graph $y_1 = 25x^2 - 20x + 4$



vertex at $\left(\frac{2}{5}, 0\right)$

"Zero" will not correctly calculate values when graph doesn't cross over x-axis ☹

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Identifying Number and Type of Solutions

- If there's $i=\sqrt{-1}$ in the answer, solutions are complex numbers.
  Complex solutions do not appear on the graph.
  ★ If solutions are complex, there are always two ±

- If there's no $i=\sqrt{-1}$ in the answer, solutions are real numbers.
  Real solutions appear as x-intercepts on graph.
  ★ If $\sqrt{0}=0$ appeared in QF work, there is only one solution
  ★ If $\sqrt{\text{positive}}$ appeared in QF work, there are two solutions. ±

- Real solutions may be rational or irrational
  Irrational solutions occur when $\sqrt{\text{positive}}$ remains in solutions.
  ★ If solutions are real and irrational, there are two solutions. ±

  Rational solutions occur when $\sqrt{\text{perfect square}}$ simplifies.
  ★ If perfect square is 0, there is only one solution.
  ★ If perfect square is not 0, there are two solutions. ±
  If solutions are rational, equations can also be solved by factoring.

In $3p^2-6p-4=0$, there are two real and irrational solutions,
$\frac{1+\sqrt{21}}{3}$ and $\frac{1-\sqrt{21}}{3}$

Number and type of Solutions (flow chart)

calculate discriminant $D = b^2 - 4ac$ (no square root)

$D\ (+)$

$\Downarrow$

Real

- Rational: $\sqrt{\text{perfect sq}} = D$
  - $\sqrt{0}$ $\Downarrow$ one real rational solution
  - other $\Downarrow$ two real rational Solutions
- Irrational: $\sqrt{\text{something that won't reduce}} = D$
  - $\Downarrow$ two real irrational solutions

$D\ (-)$

$\Downarrow$

Complex

$\Downarrow$

2 complex solutions

* Memorize Discriminant for $ax^2+bx+c=0$ is $D = b^2-4ac$ (no $\sqrt{\ }$)

e.g. $D = -23$
means
2 complex solutions

e.g $D = 23$
means
2 real irrational solutions.

e.g $D = 25$
2 real rational solutions

e.g. $D = 0$
1 real (repeated) rational solution

Without solving the equation, calculate only the discriminant and use it to determine the number and type of solutions. D= discriminant= b^2-4ac No square root!

Must be $ax^2+bx+c=0$

Extra

$$3x^2+16x+5=0$$

$D=b^2-4ac$

$16^2-4(3)(5) = \boxed{196} = 14^2$

$D>0 \Rightarrow$ 2 real solutions

D= perfect square $\Rightarrow$ $\boxed{\text{2 real, rational solutions}}$

(4) $2x^2-4x=3 \Rightarrow 2x^2-4x-3=0$

$(-4)^2-4(2)(-3)$

$\boxed{D=40}$

$D>0 \Rightarrow$ 2 real solutions

$D\neq$ perfect square $\Rightarrow$ $\boxed{\text{2 real, irrational solutions}}$

(5) $m^2-\dfrac{m}{2}+\dfrac{1}{16}=0$ clear fractions

$16m^2-8m+1=0$

$(-8)^2-4(16)(1)=\boxed{0=D}$

$D=0 \Rightarrow$ $\boxed{\text{1 real rational solution}}$

(6) $\dfrac{x}{3}=-x^2-1$ clear fractions

$x=-3x^2-1$ set $=0$

$3x^2+x+1=0$

$1^2-4(3)(1) = \boxed{-11=D}$

$D<0 \Rightarrow$ $\boxed{\text{2 complex solutions}}$

Math 70:

(9) An investment of \$2000 grows to \$2420 when compounded annually for two years. What is the interest rate?

Compounded annually $\Rightarrow$ compound interest formula

$$A = P\left(1+\frac{r}{n}\right)^{(nt)}$$

A = amount in account
P = principal, initial amount
r = annual interest rate, decimal form
t = time in years
n = # times compounded per year

$A = 2420$
$P = 2000$
$r = ?$
$t = 2$
$n = 1$

$$2420 = 2000\left(1+\frac{r}{1}\right)^{(1\cdot 2)}$$ ✓ plug in

$$2420 = 2000(1+r)^2$$ ✓ simplify $\frac{r}{1} = r$ and $1\cdot 2 = 2$

$$2000(1+r)^2 = 2420$$ ✓ swap LHS and RHS to clarify. What's left is a quadratic equation with square already completed!

$$(1+r)^2 = \frac{2420}{2000}$$

Divide both sides by 2000 to isolate the square.

$$(1+r)^2 = \frac{121}{100}$$

$\boxed{\text{MATH}}$ > frac is your friend!

$$1+r = \pm\sqrt{\frac{121}{100}}$$ square root property

$$1+r = \pm\frac{11}{10}$$

$$r = -1 \pm \frac{11}{10}$$

$$r = -1+\frac{11}{10},\quad -1-\frac{11}{10}$$

$$r = .1,\ -2.1$$

$$\boxed{r = 10\%}$$

Convert decimals to percents.
$.1 = 10\%$
$-2.1 = -210\%$

negative interest rate does not make sense. This is an extraneous solution.

Disregard -210% soln.

Rev 8-8-13

Name ____________________

Date ____________________

TI-84+ GC 26 Quadratic Formula using Memory locations, Discriminant, Graphs of Real and Complex Roots

Objectives: Review methods for solving quadratic equations
Use the discriminant to determine the nature of the solutions and usefulness of the GC
Use graph of quadratic function to discern real vs. complex roots
Perform calculations using memory locations in the GC to avoid round-off error

A quadratic equation is a degree-2 equation.

Example 1: $-3x^2+5x=7$ and $4y^2-y+1=0$ are quadratic equations.

Quadratic equations in one variable can be solved by several methods:

1. Set equal to zero, **factor**, set factors equal to zero, isolate variable for each factor equation.
2. If the equation includes a perfect square, isolate the square, and use the **square root property**.
3. Re-write by **completing the square** and use the square root property.
4. Set equal to zero and use the **quadratic formula**.
5. Approximate the solutions by **graphing** – using the intersection or x-intercept methods.

The quadratic formula $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$ is derived by completing the square for the equation $ax^2+bx+c=0$, so the quadratic formula can be used only after the equation is written in this form.

Example 2: Solve $-3x^2+5x=7$ using the quadratic formula.

Rewrite as $-3x^2+5x-7=0$, making $a=-3$, $b=5$, and $c=-7$.

Substitute these values in the quadratic formula to get the solutions: $x=\dfrac{-5\pm\sqrt{5^2-4\cdot(-3)\cdot(-7)}}{2(-3)}$.

Simplify to get $x=\dfrac{-5\pm\sqrt{-59}}{-6}$.

Write these complex solutions in the form a+bi, giving: $x=\dfrac{5}{6}\pm i\dfrac{\sqrt{59}}{-6}$ or $x=\dfrac{5}{6}\mp i\dfrac{\sqrt{59}}{6}$.

Answer: $\boxed{x=\dfrac{5}{6}\mp i\dfrac{\sqrt{59}}{6}}$

Recall: The solutions of an equation that's equal to zero are the x-intercepts or roots of the graph.

Since a graphing calculator graph shows only real (rational or irrational) numbers, a quadratic equation having complex roots will have no x-intercepts.

Important Note: Your GC will be of limited help in solving an equation if the solutions are complex or exact irrational numbers (not rounded), or if the rational result is decimal you don't recognize.

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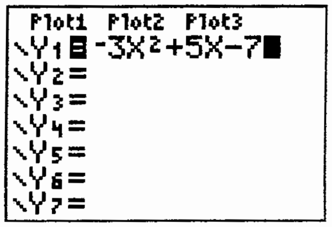
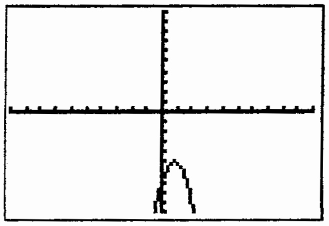
TI-84+ GC 26 Quadratic Formula using Memory locations, Discriminant, Graphs of Real and Complex Roots p.2

Example 3: Use the graph of $f(x) = -3x^2 + 5x - 7$ to determine if the roots of $-3x^2 + 5x = 7$ are real or complex.

Input $y_1 = -3x^2 + 5x - 7$ in the Y= menu and graph the parabola in a standard window. [Y=]

[CLEAR] [(−)] [3] [X,T,θ,n] [x^2] [+] [5] [X,T,θ,n] [−] [7] [ZOOM] [6]




This parabola opens downward, with its vertex below the x-axis. It has no x-intercepts. The roots are complex. Answer: <u>complex roots</u>

The <u>discriminant</u> of a quadratic equation, $D = b^2 - 4ac$, appears inside the square root of the quadratic formula.

$D < 0$ When the discriminant is a negative number, the solutions include square roots of negative numbers, which are two complex numbers.

$D \geq 0$ When the discriminant is a non-negative number, the solutions include square roots of positive numbers or zero, which are real numbers.

CAUTION: If the instructions say to calculate the discriminant, do not take the square root.

Real solutions $D \geq 0$ could have three different outcomes:
If the discriminant is a positive perfect square, the square root simplifies to a number, giving two rational solutions.
If the discriminant is zero, then the square root can be simplified to zero, giving one rational solution.
If the discriminant is positive, but not a perfect square, the square root will remain, giving irrational solutions.

Calculating the discriminant tells several things:
a) part of the solution to the quadratic equation
b) the "nature" of the solutions: real or complex, and if real, rational or irrational
c) the number of solutions: 2 complex, 2 real irrational, 2 real rational, or 1 real rational
d) whether we can use the GC to solve the equation (yes if rounded irrational, maybe if rational, no if complex or exact irrational.)

Example 4: Calculate the discriminant for $-3x^2 + 5x = 7$ and describe the nature of the solutions. Will the graphing calculator be useful in solving the equation exactly? Will your calculator be useful in solving the equation approximately?

Substitute $a = -3$, $b = 5$, and $c = -7$ into the formula for the discriminant, $D = b^2 - 4ac$.
$D = 5^2 - 4(-3)(-7) = -59$, so $D < 0$, giving two complex solutions. The GC will not be useful in solving this equation, either exactly or approximately, because the roots are complex.
Answer: <u>D= -59, 2 complex solutions, GC not useful for exact or approximate solutions.</u>

TI-84+ GC 26 Quadratic Formula using Memory locations, Discriminant, Graphs of Real and Complex Roots p.3

The GC can be very useful in calculating rounded real solutions Method 1: Use memory locations to calculate the results of the quadratic formula. Method 2: Graph the expression (after setting equal to 0) and use the Zero calculation to find the x-coordinates of the x-intercepts.

Example 5: Solve $0.127x^2 + 35.2x + 0.109 = 0$ using the quadratic formula. Round to the nearest thousandth.

Calculate the two values of x given by $x = \dfrac{-35.2 \pm \sqrt{(35.2)^2 - 4(0.127)(0.109)}}{2(0.127)}$ by using memory locations in the GC.

Store 0.127 in location A, 35.2 in location B, and 0.109 in location C.

0 . 1 2 7 STO> ALPHA MATH ENTER 3 5 .
2 STO> ALPHA APPS ENTER 0 . 1 0 9 STO> ALPHA
PRGM ENTER

```
.127→A
              .127
35.2→B
              35.2
.109→C
              .109
▮
```

$x = \dfrac{-B \pm \sqrt{B^2 - 4AC}}{2A}$ has a long fraction bar and a product in the denominator. We'll need extra parentheses, like this: $x = \dfrac{\left(-B \pm \sqrt{B^2 - 4AC}\right)}{(2A)}$. To get the first answer (plus):

((-) ALPHA APPS + 2nd x^2 ALPHA APPS x^2 − 4
ALPHA MATH ALPHA PRGM)) ÷ (2 ALPHA MATH)
ENTER

To get the second answer (minus), bring back the previous entry (2nd ENTER) and change the + to -

2nd ENTER ^ − ENTER

```
              .109
(-B+√(B²-4AC))/(
2A)
     -.0030966255
(-B-√(B²-4AC))/(
2A)
      -277.1622577
▮
```

Round answers to the nearest thousandth. Answer: <u>-0.003 and -277.162</u>

Notice: When solutions are very far from the origin, it may take a lot of work to find a graphing window that shows both x-intercepts.

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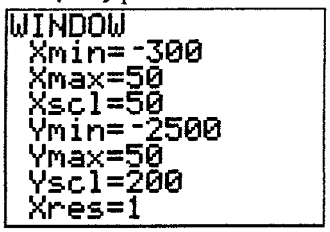
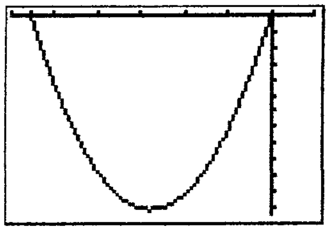
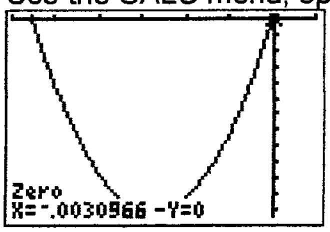
Example 6: Solve $0.127x^2 + 35.2x + 0.109 = 0$ by finding the x-intercepts. Round to the nearest thousandth. [Note: Though not done here, you could multiply by 1000 to clear the decimals.]

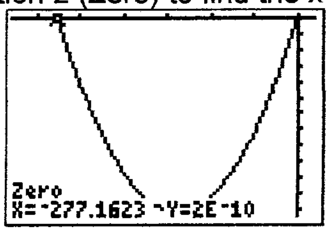
Graph $y_1 = 0.127x^2 + 35.2x + 0.109$ and adjust the window so that the x-intercepts can be seen.





Use the CALC menu, option 2 (Zero) to find the x-intercepts. Round results.



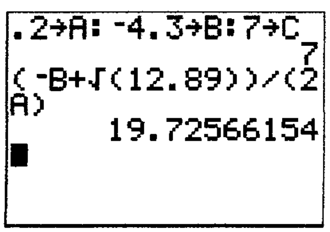


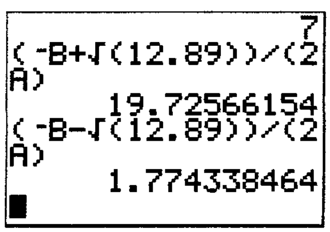
Answer: $x \approx -0.003, -277.162$

Example 7: Calculate the discriminant first, then decide whether to use your calculator and memory locations. Finally, solve $0.2x^2 - 4.3x = -7$ and round solutions to the nearest hundredth.

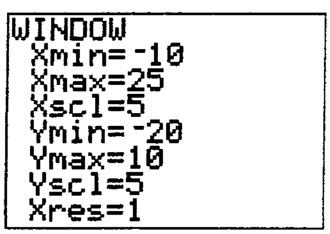
Rewrite in standard form: $0.2x^2 - 4.3x + 7 = 0$ and substitute $a = 0.2$, $b = -4.3$, and $c = 7$ in $D = b^2 - 4ac$ to get $D = (-4.3)^2 - 4(0.2)(7)$ = 12.89 D is positive but not a perfect square, so the solutions are real, irrational. Rounded real solutions can be found using the GC:

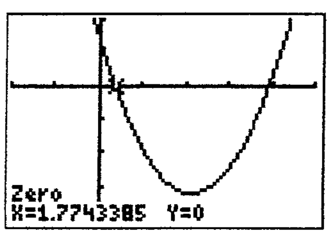
Method 1: Use memory locations and the quadratic formula.

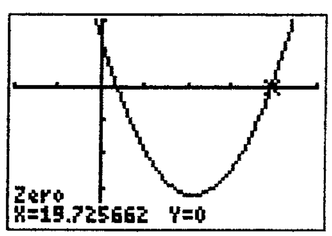




Method 2: Graph and find x-intercepts using CALC-Zero.







Answer: $D = 12.89$, $x \approx 19.73, 1.77$